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# Is the standard Higgs scalar elementary?

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## Abstract

In the standard electroweak model, the measured top quark mass requires a sizeable Yukawa coupling to the fundamental scalar. This large coupling alone might induce a dynamical breaking of the electroweak symmetry as well as non-perturbative effects. If such is the case, even a standard Higgs scalar as light as 80 GeV should have a non-negligible  $t\bar{t}$  component induced by the top condensate.

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## 1. Introduction.

Well before the advent of QCD, Nambu and Jona-Lasinio (NJL) [1] introduced a four-fermion interaction to break the chiral symmetry of strong interactions. In modern language, this NJL effective Lagrangian is expected to be induced by multiple gluon exchanges in the light quark-antiquark channels.

The electroweak gauge-couplings of the Standard Model are not strong enough to trigger a similar breaking of the flavour  $SU(2)_L \times U(1)$  gauge symmetry. But new gauge interactions beyond the Standard Model might generate effective four-fermion interactions [2]-[4]. In that approach, the scalar field  $h$  responsible for the symmetry breaking is a pure  $t\bar{t}$  composite state with  $m_h = 2m_t$ , if QCD effects are ignored.

It is however quite remarkable that the strongest force in the electroweak sector of the Standard Model is due to the Yukawa coupling of the recently observed top quark [5] to the fundamental Higgs field. It is therefore quite legitimate to investigate the possibility of a  $t\bar{t}$  condensation without having to invoke new physics beyond the Standard Model [6]. Indeed, the Yukawa coupling *itself* might generate a four-fermion interaction at some scale  $\Lambda$ , in a way similar to what is happening in QCD.

In this letter, we analyze the implications of this minimal scenario on the scalar mass spectrum. For that purpose, we assume that the  $SU(2)_L \times U(1)$  gauge couplings and the scalar self-coupling do not participate at all in the symmetry breaking. In that case, the electroweak symmetry breaking is also triggered by top quark loops [7] such that the standard Higgs boson is a linear combination of the  $t\bar{t}$  composite state and of the fundamental scalar. In particular, if the  $\Lambda$  scale is around 1 TeV, the elusive Higgs scalar is mainly a composite state with a mass of about 80 GeV.

## 2. The scalar Lagrangian.

Let us assume that the Yukawa coupling  $g_t$  of the top quark alone is indeed responsible for the electroweak symmetry breaking below the cutoff scale  $\Lambda$ . If such is the case, the relevant Lagrangian for the fundamental iso-doublet scalar field  $H$  simply becomes

$$L_H = \partial_\mu H^+ \partial^\mu H - m_H^2 H^+ H + g_t(\bar{\psi}_L t_R H + h.c.) , \quad (1)$$

where  $H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix}$  and  $\psi_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$ .

The crucial non-perturbative effect possibly induced by a large  $g_t$  Yukawa coupling is the appearance of a 4-fermion interaction

$$L_{NJL} = G\bar{\psi}_L t_R \bar{t}_R \psi_L , \quad (2)$$

after resummation of the multiple scalar exchanges in the heavy quark-antiquark channels. The 4-fermion form-factor is expected [6] to depend on the  $q^2$  transfer-momentum in the following way

$$G(q^2) = \frac{g_\sigma^2}{q^2 - m_\sigma^2} , \quad (3)$$

with  $g_\sigma$ , an effective coupling, and  $m_\sigma$ , an effective mass.

Implications of this possible non-perturbative effect are most easily described in terms of the iso-doublet auxiliary field

$$\Sigma \equiv \frac{g_\sigma}{q^2 - m_\sigma^2} \bar{t}_R \psi_L , \quad (4)$$

such that the Lagrangian

$$L_{NJL} = \partial_\mu \Sigma^\dagger \partial^\mu \Sigma - m_\sigma^2 \Sigma^\dagger \Sigma + g_\sigma (\bar{\psi}_L t_R \Sigma + h.c.) . \quad (5)$$

is equivalent to Eq.(2). The scalar Lagrangian  $L_H + L_{NJL}$  at the source of the  $SU(2)_L \times U(1)$  breaking below the scale  $\Lambda$  is then defined by Eqs.(1) and (5).

Notice that the absence of a kinetic term for the  $\Sigma$  field at the scale  $\Lambda$  would imply the compositeness boundary condition [4]  $g_\sigma(\Lambda) = \infty$  on the physically normalized coupling  $g_\sigma$ . But the coupled renormalization group equations for  $g_t$  and  $g_\sigma$  require the ratio  $g_\sigma/g_t$  to be independent of the scale. This would obviously be in contradiction with the Lagrangian given in Eq.(1) where  $g_t(\mu)$  is supposed to have a fixed and finite value at the  $\Lambda$  scale.

This kinetic term is important since it allows the  $\Sigma$  field to contribute also to the  $W$  gauge-boson mass through the covariant derivative, once the (small)  $SU(2)$  gauge coupling  $g_2$  is switched on. Therefore, both the loop-induced vacuum expectation values of the elementary field  $H^0$  and of the composite field  $\Sigma^0$  contribute to the  $W$  gauge-boson mass

$$m_W^2 = \frac{1}{2} g_2^2 (\langle H^0 \rangle^2 + \langle \Sigma^0 \rangle^2) \equiv \frac{1}{4} g_2^2 (\langle \varphi \rangle^2 + \langle \sigma \rangle^2) , \quad (6)$$

and to the top quark mass

$$m_t = g_t \langle H^0 \rangle + g_\sigma \langle \Sigma^0 \rangle \equiv \frac{1}{\sqrt{2}} (g_t \langle \varphi \rangle + g_\sigma \langle \sigma \rangle) . \quad (7)$$

### 3. The top-induced effective potential.

Now, we shall study how the electroweak symmetry is fully induced by top quark loops [7]. If we focus on the real part of the neutral  $H^0$  and  $\Sigma^0$  components in the scalar Lagrangian  $L_H + L_{NJL}$ , the one-loop effective potential reads

$$V(\varphi, \sigma) = m_H^2 \frac{\varphi^2}{2} + m_\sigma^2 \frac{\sigma^2}{2} - \frac{N_c}{8\pi^2} \int_0^{\Lambda^2} q^2 \ln \left( 1 + \frac{m_t^2}{q^2} \right) dq^2 , \quad (8)$$

with  $N_c$ , the number of colours.

The extrema conditions are given by

$$\begin{aligned} \left. \frac{\partial V}{\partial \varphi} \right|_{\langle \varphi \rangle, \langle \sigma \rangle} &= m_H^2 \langle \varphi \rangle - \frac{N_c}{8\pi^2} \sqrt{2} g_t m_t \int_0^{\Lambda^2} \frac{q^2}{q^2 + m_t^2} dq^2 = 0 , \\ \left. \frac{\partial V}{\partial \sigma} \right|_{\langle \varphi \rangle, \langle \sigma \rangle} &= m_\sigma^2 \langle \sigma \rangle - \frac{N_c}{8\pi^2} \sqrt{2} g_\sigma m_t \int_0^{\Lambda^2} \frac{q^2}{q^2 + m_t^2} dq^2 = 0 . \end{aligned} \quad (9)$$

From Eqs.(7) and (9), we obtain then the self-consistent relation

$$\frac{m_H^2 m_\sigma^2}{(g_\sigma^2 m_H^2 + g_t^2 m_\sigma^2)} = \frac{N_c}{8\pi^2} \left( \Lambda^2 - m_t^2 \ln \left( \frac{\Lambda^2}{m_t^2} + 1 \right) \right) \equiv I_1 \quad (10)$$

and the vacuum expectation values

$$\begin{aligned} \langle \varphi \rangle &= \sqrt{2} g_t \frac{m_t}{m_H^2} I_1 , \\ \langle \sigma \rangle &= \sqrt{2} g_\sigma \frac{m_t}{m_\sigma^2} I_1 . \end{aligned} \quad (11)$$

On the other hand, Eq.(6) requires the following normalization

$$v = \sqrt{\langle \varphi \rangle^2 + \langle \sigma \rangle^2} = 246 \text{ GeV} , \quad (12)$$

for these vacuum expectation values.

As it should be (see Eq.(3)), in the limit  $m_\sigma^2 \rightarrow \infty$ ,  $\langle \sigma \rangle \rightarrow 0$  and we recover the model considered in Ref.[7], with only one elementary scalar iso-doublet.

The gap equation in (10) can also be derived by requiring the existence of a Goldstone boson in the pseudoscalar neutral sector. This constraint is indeed fulfilled if the determinant of the neutral pseudoscalar squared mass matrix  $M_{PS}^2$

$$M_{PS}^2 = \begin{pmatrix} m_H^2 - g_t^2 I_1 & -g_t g_\sigma I_1 \\ -g_t g_\sigma I_1 & m_\sigma^2 - g_\sigma^2 I_1 \end{pmatrix} \quad (13)$$

is vanishing. The physical basis for the neutral pseudoscalars is obtained from the diagonalization of  $M_{PS}^2$  through a rotation of angle  $\theta_{PS}$  with

$$\tan \theta_{PS} = \frac{\langle \sigma \rangle}{\langle \varphi \rangle} . \quad (14)$$

#### 4. The standard Higgs scalar.

The squared mass matrix  $M_S^2$  for the neutral  $\varphi$  and  $\sigma$  scalar fields is obtained after the substitution  $I_1 \rightarrow I_1 - I_2$  with

$$\begin{aligned} I_2 &= \frac{N_c}{4\pi^2} m_t^2 \int_0^{\Lambda^2} \frac{q^2 dq^2}{(q^2 + m_t^2)^2} \\ &= \frac{N_c}{4\pi^2} m_t^2 \left( \ln\left(\frac{\Lambda^2}{m_t^2} + 1\right) - \frac{\Lambda^2}{\Lambda^2 + m_t^2} \right) , \end{aligned} \quad (15)$$

in the pseudoscalar squared mass matrix given in Eq.(13). Consequently, we now have to diagonalize the  $2 \times 2$  matrix

$$M_S^2 = M_{PS}^2 + \begin{pmatrix} g_t^2 & g_t g_\sigma \\ g_t g_\sigma & g_\sigma^2 \end{pmatrix} I_2 . \quad (16)$$

As  $\Lambda^2 \gg m_t^2$ ,  $I_2$  is small compared to  $I_1$  and the diagonalization angle  $\theta_S$  for the  $M_S^2$  matrix is very close to  $\theta_{PS}$ . The lightest neutral scalar field  $h$  is therefore almost in alignment with the pseudoscalar Goldstone boson:

$$h \simeq \cos \theta_{PS} \varphi + \sin \theta_{PS} \sigma . \quad (17)$$

Its squared mass proportional to  $I_2$  (see Eq.(16)) has a smooth logarithmic dependence on the scale  $\Lambda$  and is approximately given by

$$m_h^2 \approx (g_t \cos \theta_{PS} + g_\sigma \sin \theta_{PS})^2 I_2 . \quad (18)$$

The other scalar field has a mass proportional to the scale  $\Lambda$ .

From Eqs.(7), (14) and (18), we conclude that the mass  $m_h$  of the standard Higgs scalar  $h$  is equal to

$$m_h \approx \left( \frac{2I_2}{v^2} \right)^{1/2} m_t \quad (19)$$

and does not depend on its structure. In particular, it can be as small as 80 GeV if the  $\Lambda$  scale is about 1 TeV.

The interesting mass relation given in Eq.(19) is based on the assumption that the heavy top quark alone triggers the full electroweak symmetry breaking. This relation has been derived in Ref.[7] for the special case of a pure elementary scalar field ( $\theta_{PS} = 0$ ). However, Eq.(17) shows that the mass relation in Eq.(19) remains valid even if the standard Higgs scalar has a large  $t\bar{t}$  component ( $\theta_{PS} > \pi/4$ ). Such an intriguing possibility is in fact favoured if some  $t\bar{t}$  condensation takes place at the electroweak scale.

In the Standard Model without  $t\bar{t}$  condensation ( $\langle\sigma\rangle = 0$ ),  $g_t \approx 1$  corresponds to  $m_t \approx 174$  GeV. But the non-perturbative condensation mechanism ( $\langle\sigma\rangle \neq 0$ ) assumed in this letter requires a larger Yukawa coupling

$$g_t \gg 1, \quad (20)$$

at the electroweak scale. This physical constraint together with the fact that  $g_t\langle\varphi\rangle$  and  $g_\sigma\langle\sigma\rangle$  are positive (see Eqs.(11)) imply therefore

$$\langle\varphi\rangle \ll v \quad (21)$$

to reproduce the measured top quark mass defined in Eq.(7). From Eqs.(12) and (21), we then obtain the following hierarchy

$$\langle\sigma\rangle \gg \langle\varphi\rangle \quad (22)$$

among the vacuum expectation values, such that the standard Higgs scalar  $h$  defined in Eq.(17) has a dominant ( $\theta_{PS} > \pi/4$ )  $t\bar{t}$  component.

For illustration, let us assume the quite reasonable value

$$\alpha_t \equiv \frac{g_t^2}{4\pi} \simeq 1, \quad (23)$$

for the genuine Yukawa coupling of the top quark. For  $m_t = 174$  GeV, we obtain

$$\sin \theta_{PS} > 0.96 \quad (24)$$

and the standard Higgs scalar is indeed an almost pure  $t\bar{t}$  bound state.

Let us remark that for very high values of the scale  $\Lambda$ , the running of couplings down to the electroweak scale must be taken into account. If these couplings enter the perturbative regime above the electroweak scale, they will approach the quasi-fixed point [8] given in our case by

$$g_t^2 + g_\sigma^2 \approx \frac{16}{9} g_3^2, \quad (25)$$

with  $g_3$ , the strong interaction coupling. The existence of such a point usually leads to a too heavy top quark in top condensate models [4]. However, in the present case, the top quark mass is a linear combination of the  $g_t$  and  $g_\sigma$  couplings and its experimental value [5] can be reproduced even if the quasi-fixed point is reached.

We emphasize however that, in the approach considered here, there is no reason whatsoever to have a very high scale  $\Lambda$  to reproduce the  $W$  gauge-boson mass (see Eq.(6)).

## 5. Conclusion

Assuming that a strong Yukawa coupling of the top quark is fully responsible for the electroweak symmetry breaking [6, 7], we have shown that a  $t\bar{t}$  condensation would imply a large composite component for a rather light standard Higgs scalar. The effective approach presented in this letter illustrates how a dynamical symmetry breaking might, in principle, avoid two generic problems associated with top condensate models [2]-[4], namely a (too) heavy top quark due to compositeness boundary conditions and a (too) high  $\Lambda$  scale due to loop-induced gauge-boson masses. Here, the genuine top Yukawa coupling must be finite at the  $\Lambda$  scale and the gauge-boson masses are induced by  $SU(2) \times U(1)$  covariant derivatives. It is also remarkable that the mass prediction of Ref.[7] for the Higgs scalar remains valid in the presence of a top condensate. In particular, the usual relation  $m_H \approx 2m_t$  does not apply and a very light Higgs scalar is predicted if  $\Lambda$  is around 1 TeV.

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